A Report of Experiments on the Best-of-Many Christofides Algorithms and Ant Colony Optimization for the Traveling Salesman Problem

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Abstract. *This paper aims at evaluating different approaches to approximately solve the Traveling Salesman Problem (TSP). In the light of recent research based on the well-known Christofides' Algorithm, new variations have emerged. These are the so called Best-of-Many (BoM) Christofides' Algorithms, which provide better results: if not in guarantees of a better approximation ratio, at least they show better performance in a quantitative analysis. Work in the literature can be found that have empirically compared the original Christofides' Algorithm with the well-known Ant Colony Optimization (ACO) Metaheuristic. We have made experiments comparing ACO and the BoM Christofides' Algorithms, finding that the latter ones have demonstrated far better performance for the instances that we have considered.*

1. Introduction

The General Traveling Salesman Problem (GTSP) consists of, given a graph $G = (V, E)$, finding a tour $T = v_0 e_1 v_1 \dots e_{n-1} v_{n-1} e_n v_0$, wherein $\{v_0, \dots, v_{n-1}\} = V$ and $\{e_1, \dots, e_n\} \subseteq E$, which visits every vertex in the graph exactly once, such that the sum of the costs $c(e_i)$ of the edges e_i in T is minimum among all possible such tours in the graph. The problem is known to be NP-Hard [Karp 1972]; being so, there is no exact algorithm that can provide a solution within a bounded polynomial upper limit, unless $\mathsf{P} = \mathsf{NP}$. Several approximation algorithms have been developed to provide solutions that are within a factor of approximation of the optimal, and currently the best is provided by [Christofides 2022], achieving a ratio of $3/2$, meaning that any solution to the problem is at most 50% worse than the optimal solution. For this approximation, restrictions must be made on the problem, namely (for all $i, j, k \in V$):

- the costs on the graph are symmetric (i.e. $c(i, j) = c(j, i)$);
- all costs are non-negative;
- the costs must obey the triangle inequality (i.e. $c(i, j) + c(i, k) \geqslant c(j, k)$).

These restrictions on the problem define the *Metrical TSP* and will be assumed whenever referring to the TSP from now on. The idea behind Christofides' Algorithm, briefly speaking, is that taking the minimum spanning tree of a graph can help us get a good starting point to find a tour. Computing a minimum-cost perfect matching on the odd-degree vertices of the spanning tree, we have a graph that contains all vertices of the original and some of the edges. Since this graph has only even-degree vertices, we can quickly compute an Eulerian cycle and then shortcut the tour on our graph by removing edges that connect already visited vertices.

Work from the literature have been made aiming at empirically compare the original Christofides' Algorithm with the well-known Ant Colony Optimization (ACO)

Metaheuristic, e.g. [Cheong et al. 2017]. This metaheuristic simulates the behavior of ant colonies trying to find food and bring back to the nest, while they leave behind pheromones that get stronger or weaker depending on how popular the paths are. Although under no theoretical guarantee of an approximation ratio, this metaheuristic have been extensively used for TSP in practice [Ibid.].

We present in Sect. 3 a report of experiments comparing ACO and the Best-of-Many Christofides' Algorithms, new variants of the Christofides' Algorithm which have emerged in the last decade which are briefly presented in Sect. 2.

2. Best-of-Many (BoM) Christofides' Algorithms

A brief explanation of the approaches used follow:

- Maximum Entropy Distribution (MaxEnt) [Asadpour et al. 2017] randomly selects a spanning tree sampled from a maximum entropy distribution. An interesting fact about this randomized algorithm is that, considering the harder Asymmetric TSP, it achieves a $O(\log n/\log\log n)$-approximation ratio with high probability.
- Column Generation Algorithm (ColGen) [An 2012] generates spanning trees in convex combinations, using increasingly more trees while trying to minimize the cost of the tour. Considering the s–t-path variant of the TSP, this algorithm achieves a $((1+\sqrt{5})/2)$-approximation ratio.
- In Splitting off (Split), it is computed "a packing of spanning trees via iteratively *splitting off* edges of the LP solution from vertices, then maintaining a convex combination of trees as we *lift back* the split-off edges" [Genova and Williamson 2015].
- Swap Round (SR) [Chekuri and Vondrák 2009] is used in tandem with the previous two algorithms, defining a strategy to select a spanning tree from a negatively correlated distribution to the convex combination created by the other algorithms.

3. Results

Our evaluation was based on the previous research by [Genova and Williamson 2015] that set to compare the performance of different implementations of the Best-of-Many Christofides Algorithms. Adding to these implementations, we have decided to compare the performance of these approximation algorithms to an Ant Colony Optimization metaheuristic, based of the original implementation of [Dorigo et al. 2006]. We have run all the BoM algorithms implemented by Genova and Williamson against a set of instances, and for each instance we have run ACO for as long as the BOM algorithm that took the longest to complete execution. The empirical results are reported in Fig. 1 and Table 1. In our report, BoM Algorithms are identified by the same acronyms introduced in Sect. 2, while the standard Christofides' Algorithm is identified simply by "Std". The full data obtained can be found in the following URL:

`https://github.com/Newmaker0/Results-ACO-BOMC-TSP`

We have been able to evaluate that the ACO was outperformed by the BoM algorithms for all instances, and significantly so in larger instances, where we could observe a much higher approximation factor. There can be made an argument about this

Figure 1. Approximation factor for BOM algorithms

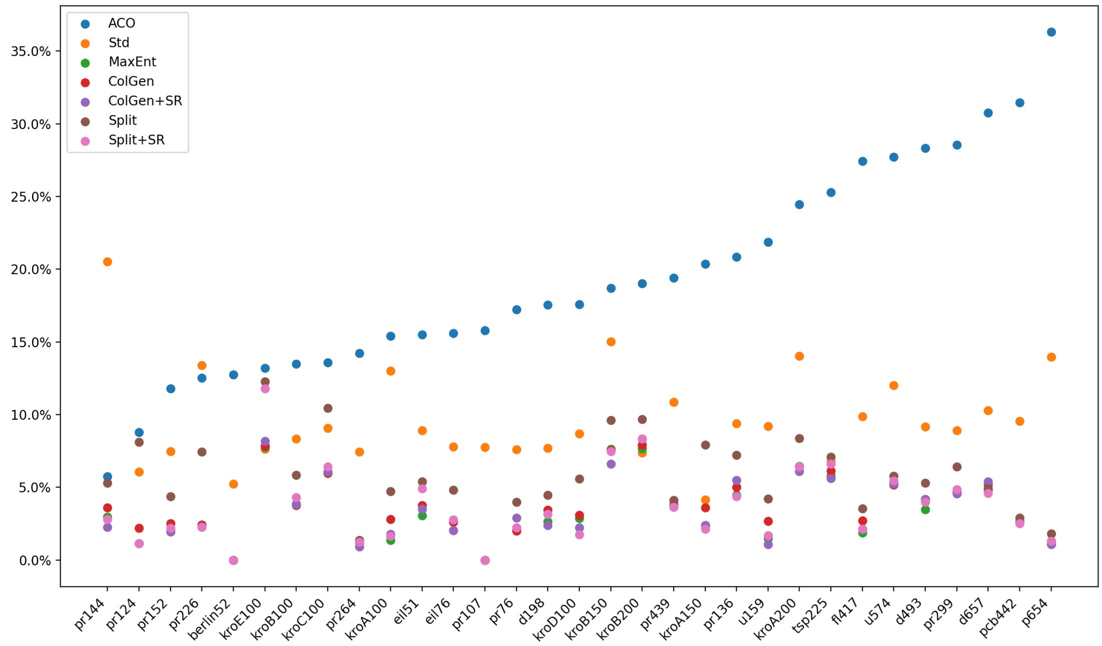

Table 1. Average approximation factor

	ACO	BOM	Std	MaxEnt	ColGen	ColGen+SR	Split	Split+SR
Averages	19.4%	3.97%	9.59%	3.39%	3.75%	3.43%	5.59%	3.68%

phenomenon and we hypothesize that it may be caused by the convergion of the meta heuristic to local optima. We encourage future investigation on experiments considering these efficient implementations of BoM algorithms and more improved implementations of ACO.

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